

4期生数学サブゼミテスト⑱

About 80 minutes

(200 points)

NOTICE

- 1 All the numbers in this test are real number, if there is no notice.
- 2 Hurry up as fast as possible.
- 3 If you have any questions, raise your hand quietly and let officer know.
- 4 You can use pencil and ruler.
- 5 You must answer on “answer sheet” differentiated from “question sheet”. If you answer on “question sheet”, you will get no score with the answers.
- 6 Write your name at the top space on the answer sheet.
- 7 You can also get some scores from the process of answering.
In other word, you must write the process.

Answer in Japanese or English. Good Luck!

1. What is the condition that $\frac{a}{b}$ is able to exist?
2. Define function with independent variable and dependent variable.
3. Explain the difference between variable and parameter.
4. Solve the following problem about geometric progression $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots\}$.
 - (1) general member a_n
 - (2) the sum from first member to “n” member S_n
 - (3) the sum from first member to “ ∞ ” member S_∞
5. Solve the inequality: $x^2 - 8x + 15 > 0$
6. Solve the equation: $2x^2 - x - 6 = 0$
7. Draw the function: $y = x^2 - 2x + 3$
8. Draw these following functions.
 - (1) $y = \frac{2x-1}{x+1}$
 - (2) $y = \sqrt{2x+4}$
9. Find inverse function and domain of function $y = \frac{2x+3}{x+1}$.

10. Draw the territory of the following coalition inequality: $y < x^2$ and $y > 2x + 3$.

11. Assume $A = \begin{pmatrix} 2 & 4 \\ 1 & -3 \end{pmatrix}$, $B = \begin{pmatrix} 8 & 5 \\ 0 & 1 \end{pmatrix}$. Evaluate $2A + 5B$.

12. Evaluate $\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

13. Evaluate the inverse matrix of $\begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}$

14. There is the dart.

The following table shows the point and the probability of the point.

Point	0	5	10
Probability	1/2	1/3	1/6

Evaluate expectation, variance and standard deviation.

15. Define range and domain.

16. $f(x) = x^2$. Show $\frac{d}{dx} f(x) = 2x$ with $\frac{d}{dx} f(x) \equiv \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$.

17. Differentiate the function with respect to x .

(You do not use derivation definition: $\frac{d}{dx} f(x) \equiv \lim_{dx \rightarrow 0} \frac{f(x + dx) - f(x)}{dx}$.)

$$\cdot f(x) = \sqrt{x} - \frac{1}{3x} + 1$$

18. Obtain the tangential line of the point on the function: $(x,y)=(1,2)$ on $y = 2x^3$.

19. Draw the function on xy plane: $f(x) = -x^3 + 12x$

20. Multiply out $\sum_{k=1}^n \sum_{i=1}^k nk$.

21. What does it tell us about the function whether the slope of the tangent line along a function is plus or not?

22. What is the signification of the second derivative?

23. Define differentiable function.

24. Are the function $(x \rightarrow y)$ differentiable in respect to x at the point?

Verify and tell the reason by the algebra and the graph on the function.

$$\bullet \ y = \begin{cases} x^2 - 8x + 14 & (x \geq 2) \\ x^2 - 2 & (x < 2) \end{cases} \quad \text{at } (2,2)$$

25. Define exponential.

26. Show the graph of $y = a^x (a > 1)$

27. Define logarithms with $y = \log_a x$.

28. Show the graph of $y = \log_a x (a > 1)$

29. What is the inverse function of $y = a^x$? Show it by using algebra and graph.