４期生数学サブゼミテスト⑰

About 30 minutes

NOTICE

1. All the numbers in this test are real number, if there is no notice.
2. Hurry up as fast as possible.
3. If you have any questions, raise your hand quietly and let officer know.
4. You can use pencil and ruler.
5. You must answer on “answer sheet” differentiated from “question sheet”. If you answer on “question sheet”, you will get no score with the answers.
6. Write your name at the top space on the answer sheet.
7. You can also get some scores from the process of answering. In other word, you must write the process.

Answer in Japanese or English. Good Luck!

1. Cobb-Douglas utility function is $U = x^{\frac{1}{3}} y^{\frac{2}{3}} \, (x, y \to U \mid R^2 \to R)$.
   Represent percent change of utility with percent change of x and that of y.

2. Demand function is $Q = \dfrac{2}{P} \, (P \to Q \mid R \to R)$. Make it into linear function and represent price elasticity of demand.

3. Production function is $Y = AK^{0.3} L^{0.7} \, (K, L \to Y \mid R^2 \to R)$.
   Represent percent change of Y with percent change of K, that of L and that of A.

Go to the next page.

4. Fill the blanks.

Proposition: $\frac{d}{dx}\log_a x = \frac{1}{x\ln a}$

Proof:

$$\frac{d}{dx}\log_a x = \lim_{h\to 0}\frac{<1>-<2>}{h} = \lim_{h\to 0}\frac{\log_a <3>}{h} = \lim_{h\to 0} <4> \log_a <5>$$

$$= \lim_{h\to 0}\log_a <5>^{<4>} \qquad \ldots①$$

Definition: $t \equiv \frac{h}{x} \quad \therefore \quad h \to 0 \ \Rightarrow \ t \to 0 \ \text{ and } \ t \equiv \frac{h}{x} \quad \Rightarrow \quad \frac{1}{h} = \frac{1}{xt}$

$$\frac{d}{dx}\log_a x = \lim_{h\to 0}\log_a <5>^{<4>} = \lim_{<6>}\log_a <7>^{<8>} = \lim_{<6>}\log_a <7>^{<8>\cdot<9>}$$

$$= \lim_{<6>} <9> \log_a <7>^{<8>}$$

x is independent of t, so

$$\frac{d}{dx}\log_a x = <9> \lim_{<6>}\log_a <7>^{<8>} = <9> \log_a\left\{\lim_{<6>} <7>^{<8>}\right\}$$

Because $e = \lim_{<6>} <7>^{<8>}$, $\frac{d}{dx}\log_a x = <9> \log_a <10>$

$$\frac{d}{dx}\log_a x = <9> \frac{\log_e <11>}{\log_e <12>} = \frac{<13>}{<14> \log_{<15>} <16>}$$

$$\therefore \quad \frac{d}{dx}\log_a x = \frac{1}{x\ln a} \qquad \text{(Q.E.D.)}$$